CTP TAMU-49/95
hep-th/9512082
December 1995

Super p-Form Charges and a Reformulation of the Supermembrane Action in Eleven Dimensions *

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ABSTRACT

We discuss an extension of the super-Poincaré algebra in $D = 11$ which includes an extra fermionic charge and super two-form charges. We give a geometrical reformulation of the $D = 11$ supermembrane action which is manifestly invariant under the extended super-Poincaré transformations. Using the same set of transformations, we also reformulate a superstring action in $D = 11$, considered sometime ago by Curtright. While this paper is primarily a review of a recent work by Bergshoeff and the author, it does contain some new results.

* Talk presented at the Workshop on *Gauge theories, applied supersymmetry and quantum gravity*, Leuven, Belgium, July 10-14, 1995.

† Research supported in part by NSF Grant PHY-9411543

1 Introduction

Developments in the study of nonperturbative string physics over the last five years have revealed the importance of super p–branes. In particular, there are tantalizing hints at the possibility of an important role to be played by the eleven dimensional supermembrane theory. There is growing evidence for the existence of a fundamental theory in eleven dimensions which is described by the eleven dimensional supergravity in a certain low energy limit [1]. At present, the only available theory of extended objects that produces correctly the eleven dimensional supergravity seems to be the eleven dimensional supermembrane theory [2]. Thus, it is natural to probe further into the structure of this remarkably unique theory.

One aspect of the $D = 11$ supermembrane theory which is certainly worth exploring is the structure of its underlying spacetime supersymmetry algebra. The purpose of this paper is to summarize some recent results obtained by Bergshoeff and the author [3] in this area, and along the way, to provide a modest extension of those results. To be more specific, we will describe the extension of the supersymmetry algebra by the inclusion of an extra fermionic charge and super two-form charges. The latter are motivated by the current algebra that emerges in study of the known supermembrane action. Using the extended supersymmetry algebra, we shall provide a geometrical reformulation of the supermembrane action that generalizes a similar result obtained by Siegel [4] for the Green-Schwarz superstring. In fact, a further generalization of our $D = 11$ supersymmetry algebra which includes also super five-form charges seems to be possible. We will give the details of this extension elsewhere.

The super two-form and five-form charges occurring in the extended supersymmetry algebra are intimately connected with the existence of supermembrane [5] and super five-brane [6] solitons of $D = 11$ supergravity, while the role of the extra fermionic generator (which can be vied as part of a super one-form generator) seems less clear at present. One might think that it should play a role in the construction of a string theory in eleven dimensions. Indeed, sometime ago Curtright [7] constructed a superstring action in $D = 11$. However, the action lacked κ–symmetry, and it was not clear if it described a physically viable theory. Although this still remains to be the case, we will nonetheless reformulate Curtright's action in a way which makes use of the extended super-Poincaré algebra considered here.

An important motivation for our work is the search for a covariant supersymmetric action that would describe a super five-brane in eleven dimensions. A related question is how to describe (preferably in a covariant fashion) the dynamics of super p–brane solitons that arise in supergravity theories in diverse dimensions [8]. Thus, before presenting our results, we shall first go over various aspects of this problem.

We begin by recalling that super p-brane solutions of supergravity theories seem to be inevitable. For example, in addition to the membrane and five-brane solitons of $D = 11$ supergravity mentioned above, the heterotic string has string and five-brane solitons, Type IIA supergravity in $D = 10$ has p-brane solitons with $p = 0, 2, 4, 6, 8$ and Type IIB supergravity in $D = 10$ has p-brane solitons with $p = -1, 1, 3, 5, 7, 9$ ($p = -1$ corresponding to instantons).

The super p-brane solitons typically preserve half of the supersymmetries and give rise zero modes that form matter supermultiplets on a $(p+1)$-dimensional world-volume. One then expects a Nambu-Goto type supersymmetric actions to describe the dynamics of the zero-mode fields corresponding to the physical degrees of freedom propagating on the worldvolume. A natural way to covariantize this action [9] is to introduce worldvolume reparametrization and κ symmetries, which then makes it possible to make the target space Lorentz and supersymmetries manifest. However, that is not always a straightforward procedure, if possible at all.

Indeed, the first super p-brane action (with $p > 1$) was constructed in [9] in a manner described above. A three-brane soliton of $N = 2, D = 6$ super-Yang-Mills was considered, and a covariant action describing a super three-brane in six dimensions was obtained. Later, this result was generalized to other p-branes in various dimensions [2], without any reference to p-brane solitons, but solely on the basis of symmetries. While the case of supermembrane in $D = 11$ was emphasized in [2], actions and the necessary constraints on the spacetime background were given for all super p-branes as well. Later, it was shown [10] that these constraints restricted the possible values of (p, D) as follows: $(p = 1; D = 3, 4, 6, 10)$, $(p = 2; D = 4, 5, 7, 11)$, $(p = 3; D = 6, 8)$, $(p = 4; D = 9)$ and $(p = 5; D = 10)$. A common feature of these theories is that in a physical gauge, the physical degrees of freedom on the world-volume form scalar supermultiplets.

With the emergence of new kinds of super p-brane solitons, it became clear that there were other possible supermultiplets of zero-modes on the world-volume. For example, a five-brane soliton in Type IIA supergravity admits the $(1,1)$ supersymmetric Maxwell multiplet, and a five-brane soliton in $D = 11$ supergravity admits the $(2,0)$ supersymmetric antisymmetric tensor multiplet in $D = 6$, as zero-mode multiplets (see [8], for a review). In fact, one can turn the argument around and conjecture the existence of super p-brane solitons for every possible matter supermultiplets in $p+1$ dimensions that contains n scalar fields. Assuming that the sacalars correspond to translational zero-modes, it follows that the dimension D of the target space is $D = n + p + 1$. These leads to a revised p-brane scan [11].

It is useful to make a distinction between super p-branes according to the nature of the word-volume degrees of freedom. We will tentatively refer to them as *scalar*, *vector* and *tensor p-branes*, depending on whether the world-volume physical degrees of freedom form scalar, vector or

antisymmetric tensor supermultiplets.

The problem of how to describe the dynamics of super p–branes of the type mentioned above seemed rather unsurmountable in the past. However, with recent advances in the studies of duality symmetries in nonperturbative string theory, the prospects of finding a suitable framework for super p–brane dynamics look brighter. In particular, the idea of Dirichlet p–branes [12, 13] is a very useful one in that it seems to provide a conformal field theoretic stringy description of the p-branes, albeit in a physical gauge. In this approach, one considers open strings whose boundaries are constrained to move on $(p+1)$ dimensional plane, which has its own dynamics, and therefore it can be interpreted as the world-volume of a p-brane. However, this description seems capable of describing only the scalar and vector p–branes, but not the tensor p–branes. Perhaps a generalization of the ideas of [13] can also lead to a stringy description of tensor p–branes. This is an open problem.

Another open problem along the lines discussed above is how to find a *covariant* description of the vector and tensor p–branes. In this note, we will summarize some ingredients which might play a useful role in achieving this. To provide a further background to our work, let us recall that the original covariant p-brane actions of [2] describe the scalar p–branes and that they are essentially sigma models with Wess-Zumino terms where the coordinates fields map a bosonic $(p+1)$–dimensional world-volume M into a target superspace N in D–dimensional spacetime. This suggest a generalization where one considers more general M and N. Indeed, models have been constructed where M itself is elevated into a superspace. For a formulation of the eleven dimensional supermembrane in such a formalism, see [14]. These formulations are rather complicated, however, so much that it is a nontrivial matter even to show exactly what the physical degrees of freedom are.

A rather simple reformulation of the superstring [4] and the super p-branes [3] does exist, however. In this approach, one leaves the world-volume as bosonic but generalizes the target superspace in such a way that the target space super Poincaré algebra is extended to include new bosonic and fermionic generators which are motivated by the existence of super p-branes in those dimensions. Such generalizations of the superspace are then hoped to open new avenues to describe new degrees of freedom. In particular, it is rather suggestive that the new coordinates introduced in this fashion may play an important role in the description of duality symmetries in super p-brane theories– a subject of great potential importance.

With the above motivations in mind, we now proceed to summarize more explicitly the main ideas involved in the description of the new types of super p-brane actions, based on new types of target space superalgebras. Although we will focus our attention exclusively on the extension of the super-Poincaré algebra and the supermembrane in $D = 11$, the ideas of this paper can easily

be applied to the other super p–branes as well [3].

2 Extension of the $D = 11$ Super Poincaré Algebra

The elements of the ordinary super Poincaré algebra in $D = 11$ are the translation generator P^μ ($\mu = 0, 1, ..., 10$) and the supersymmetry generators Q_α ($\alpha = 1, ..., 32$). Let us consider the extra generators Σ^α, $\Sigma^{\mu\nu} = -\Sigma_{\nu\mu}$, $\Sigma^{\mu\alpha}$ and $\Sigma^{\alpha\beta} = \Sigma^{\beta\alpha}$. We can show that the following extension of the $D = 11$ super Poincaré algebra exists

$$\begin{aligned}
\{Q_\alpha, Q_\beta\} &= \Gamma^\mu_{\alpha\beta} P_\mu + (\Gamma_{\mu\nu})_{\alpha\beta}\Sigma^{\mu\nu} \ , \\
[Q_\alpha, P_\mu] &= (\Gamma_\mu)_{\alpha\beta}\Sigma^\beta + (\Gamma_{\mu\nu})_{\alpha\beta}\Sigma^{\nu\beta} \ , \\
[P_\mu, P_\nu] &= (\Gamma_{\mu\nu})_{\alpha\beta}\Sigma^{\alpha\beta} \ , \\
[P_\mu, \Sigma^{\lambda\tau}] &= \tfrac{1}{2}\delta_\mu^{[\lambda}\Gamma^{\tau]}_{\alpha\beta}\ \Sigma^{\alpha\beta} \ , \\
[Q_\alpha, \Sigma^{\mu\nu}] &= -\tfrac{1}{10}\Gamma^{\mu\nu}_{\alpha\beta}\Sigma^\beta + (\Gamma^{[\mu})_{\alpha\beta}\Sigma^{\nu]\beta} \ , \\
\{Q_\alpha, \Sigma^{\nu\beta}\} &= \left(\tfrac{1}{4}\Gamma^\nu_{\gamma\delta}\delta^\beta_\alpha + 2\Gamma^\nu_{\gamma\alpha}\delta^\beta_\delta\right)\Sigma^{\gamma\delta} \ ,
\end{aligned} \tag{2.1}$$

with all the other (anti) commutators vanishing. The generators $\Sigma^{\alpha\beta}$ and Σ^α are central, and they can be contracted away. In fact, setting Σ^α equal to zero yields an algebra proposed recently in [3], and setting both, S^α and $\Sigma^{\alpha\beta}$ equal to zero yields an algebra proposed sometime ago in [16]. A dimensional reduction of the algebra (2.1 Extension of the $D = 11$ Super Poincaré Algebraequation.2.1) to $D = 10$, followed by a chiral truncation and a suitable contraction, yields an algebra that contains P_μ, Q_α and Σ_α, found sometime ago by Green [15].

The usual Lorentz algebra can be incorporated into (2.1 Extension of the $D = 11$ Super Poincaré Algebraequation.2.1), without spoiling the Jacobi identities. As the algebra (2.1 Extension of the $D = 11$ Super Poincaré Algebraequation.2.1) is already closed, we shall not consider the Lorentz generators any further.

To verify the Jacobi–identities one needs the following Γ–matrix identities

$$\begin{aligned}
&\Gamma^{\mu\nu}_{(\alpha\beta}\Gamma^\nu_{\gamma\delta)} = 0 \ , \\
&\Gamma^\mu_{(\alpha\beta}\Gamma^\mu_{\gamma\delta)} - \tfrac{1}{10}\Gamma^{\mu\nu}_{(\alpha\beta}\Gamma^{\mu\nu}_{\gamma\delta)} = 0 \ .
\end{aligned} \tag{2.2}$$

(Note that, the second identity follows from the first one).

The super two-form Σ generators introduced above are motivated by the structure of a super-current algebra that arises in the study of charge density currents in the known supermembrane action [16]. One expects that suitable soliton solutions of $D = 11$ supergravity to carry the Σ charges [18]. Since, in $D = 11$ also a super fivebrane soliton exists [6], it is natural to include

the fifth rank generators $\Sigma^{\mu_1\cdots\mu_5}$, $\Sigma^{\mu_1\cdots\mu_4\alpha_5}$,..., $\Sigma^{\alpha_1\cdots\alpha_5}$. In fact, we have have already succeeded in including the $\Sigma^{\mu_1\cdots\mu_5}$ and $\Sigma^{\mu_1\cdots\mu_4\alpha_5}$, and a further inclusion of the remaining Σ generators should be possible. For the purposes of this note, we shall leave out the super five-form Σ generators, which we shall treat elsewhere.

It should be mentioned that extensions of the super-Poincaré algebra in eleven dimensions have been considered before, but they differ from (2.1 Extension of the $D = 11$ Super Poincaré Algebraequation.2.1). One such extension was given by van Holten and van Proeyen [17]. Their algebra corresponds to $OSp(32|1)$. It contains only 32 fermionic generators Q_α and all the bosonic generators correspond to the decomposition of $Sp(32)$ with respect to $SO(10,1)$. A suitable contraction of this algebra reduces to an algebra with only Q_α, P_μ and $\Sigma_{\mu_1\ldots\mu_5}$ kept. In [17], also a Lorenzian decomposition of $OSp(64|1)$ was considered. It contains 64 fermionic generators, and it also differs from our algebra. In fact, it should be stressed that no connection is known at present between the algebra (2.1 Extension of the $D = 11$ Super Poincaré Algebraequation.2.1) and any contraction of the classified simple or semi-simple Lie superalgebras. It would be interesting to find such a connection.

Another extension of the eleven dimensional algebra was considered by D'Auria and Fré [19], in their geometrical formulation of the eleven dimensional supergravity. Their algebra contains the generators Σ^α, $\Sigma^{\mu\nu}$ and $\Sigma^{\mu_1\cdots\mu_5}$, in addition to P_μ and Q_α. It can be viewed as contraction of an extended version of (2.1 Extension of the $D = 11$ Super Poincaré Algebraequation.2.1) that was mentioned above.

Finally, a version of the algebra (2.1 Extension of the $D = 11$ Super Poincaré Algebraequation.2.1) where only P^μ, Q^α and $\Sigma_{\mu_1\ldots\mu_5}$ are kept, has also arisen in [18], in the context of a topological extension of the superalgebras for extended objects. It would be interesting to extend the work of [18] to seek supermembrane configurations which will carry not only the bosonic charge $\Sigma^{\mu\nu}$, but all the charges occurring in (2.1 Extension of the $D = 11$ Super Poincaré Algebraequation.2.1).

Let us now turn to a more detailed discussion of the algebra (2.1 Extension of the $D = 11$ Super Poincaré Algebraequation.2.1). It is useful to introduce the following notation for the generators

$$T_A = \Big(P_\mu, Q_\alpha, \Sigma^\alpha, \Sigma^{\mu\nu}, \Sigma^{\mu\alpha}, \Sigma^{\alpha\beta}\Big)\ , \tag{2.3}$$

and write the superalgebra as $[T_A, T_B\} = f_{AB}{}^C\, T_C$. The structure constants $f_{AB}{}^C$ can be read off from (2.1 Extension of the $D = 11$ Super Poincaré Algebraequation.2.1). Using these structure constants, one finds that the Cartan-Killing metric vanishes:

$$\mathrm{Str}_{\,\mathrm{adj}}(\mathrm{T_A T_B}) = 0\ . \tag{2.4}$$

Presumably, this does not rule out the existence of a suitable nondegenerate metric, which, however, remains to be constructed.

A suitable parametrization of the supergroup manifold based on the algebra (2.1 Extension of the $D = 11$ Super Poincaré Algebraequation.2.1) takes the form

$$U = e^{\phi_{\mu\nu}\Sigma^{\mu\nu}}\ e^{\phi_{\mu\alpha}\Sigma^{\mu\alpha}}\ e^{\phi_{\alpha\beta}\Sigma^{\alpha\beta}}\ e^{\phi_\alpha\Sigma^\alpha}\ e^{x^\mu P_\mu}\ e^{\theta^\alpha Q_\alpha}\ , \tag{2.5}$$

where we have introduced the coordinates

$$Z^M = (x^\mu, \theta^\alpha, \phi_\alpha, \phi_{\mu\nu}, \phi_{\mu\alpha}, \phi_{\alpha\beta})\ . \tag{2.6}$$

We can define left–invariant currents ${L_i}^A$ and right–invariant currents ${R_i}^A$ as usual:

$$\begin{aligned} U^{-1}\partial_i U &= \partial_i Z^M {L_M}^A T_A = {L_i}^A T_A\ , \\ \partial_i U U^{-1} &= \partial_i Z^M {R_M}^A T_A = {R_i}^A T_A\ . \end{aligned} \tag{2.7}$$

The Maurer–Cartan forms $L^A = dZ^M {L_M}^A$ and $R^A = dZ^M {R_M}^A$ obey the structure equations $dL^A - \frac{1}{2}L^B \wedge L^C\, {f_{CB}}^A = 0$ and $dR^A + \frac{1}{2}R^B \wedge R^C\, {f_{CB}}^A = 0$. (We are using the conventions of [20]). In particular, we have the Cartan integrable system

$$\begin{aligned} dL^\alpha &= 0\ , \qquad dL^\mu = \tfrac{1}{2}L^\alpha \wedge L^\beta \Gamma^\mu_{\alpha\beta}\ , \\ dL_\alpha &= L^\mu \wedge L^\beta (\Gamma_\mu)_{\alpha\beta} - \tfrac{1}{10}L_{\mu\nu} \wedge L^\beta \Gamma^{\mu\nu}_{\alpha\beta}\ , \\ dL_{\mu\nu} &= \tfrac{1}{2}L^\alpha \wedge L^\beta (\Gamma_{\mu\nu})_{\alpha\beta}\ , \\ dL_{\mu\alpha} &= L^\beta \wedge L^\nu (\Gamma_{\mu\nu})_{\beta\alpha} + L^\beta \wedge L_{\mu\nu}\Gamma^\nu_{\beta\alpha}\ , \\ dL_{\alpha\beta} &= -\tfrac{1}{2}L^\mu \wedge L^\nu (\Gamma_{\mu\nu})_{\alpha\beta} + \tfrac{1}{2}L_{\mu\nu} \wedge L^\mu \Gamma^\nu_{\alpha\beta} + \tfrac{1}{4}L_{\mu\gamma} \wedge L^\gamma \Gamma^\mu_{\alpha\beta} + 2L_{\mu(\alpha} \wedge L^\gamma \Gamma^\mu_{\beta)\gamma}\ . \end{aligned} \tag{2.8}$$

The supergroup generators can be realized as the right–translations on the group given by $T_A = {R_A}^M \partial_M$, while the supercovariant derivatives invariant under these transformations can be realized as in terms of the left–translations as $D_A = {L_A}^M \partial_M$, where ${R_A}^M$ and ${L_A}^M$ are the inverses of ${R_M}^A$ and ${L_M}^A$, respectively. The supergroup transformations can be written as

$$\delta Z^M = \epsilon^A {R_A}^M\ , \tag{2.9}$$

where ϵ^A are the constant transformation parameters. The explicit expressions for ${L_i}^A$ and δZ^M (with $\Sigma^\alpha = 0$) can be found in [3]. In particular, including the S^α generator, one finds that [1]

$$L_i^\alpha = \partial_i \theta^\alpha\ , \qquad L_i^\mu = \partial_i x^\mu + \tfrac{1}{2}\bar\theta\Gamma^\mu\partial_i\theta\ ,$$

[1] In the calculations we never need to raise or lower a spinor index using the charge–conjugation matrix. It is convenient to use a notation where a given spinor always has an upper or a lower spinor–index, e.g. $Q_\alpha, \Sigma^{\mu\beta}, \theta^\alpha$, etc. In case we do not denote the spinor indices explicitly, it is always understood that they have their standard position, e.g. $(\Gamma_\mu\theta)_\alpha = (\Gamma_\mu)_{\alpha\beta}\theta^\beta, \bar\theta\Gamma^\mu\partial_i\theta = \theta^\alpha(\Gamma^\mu)_{\alpha\beta}\partial_i\theta^\beta$, etc.

$$
\begin{aligned}
L_{i\alpha} &= \partial_i\phi_\alpha - \partial_i x^\mu \left(\Gamma_\mu\theta\right)_\alpha + \tfrac{1}{10}\partial_i\phi_{\mu\nu}\left(\Gamma^{\mu\nu}\theta\right)_\alpha - \tfrac{1}{6}\left(\Gamma_\mu\theta\right)_\alpha \bar\theta\Gamma^\mu\partial_i\theta \ , \\
L_{i\mu\nu} &= \partial_i\phi_{\mu\nu} + \tfrac{1}{2}\bar\theta\Gamma_{\mu\nu}\partial_i\theta \ , \\
L_{i\mu\alpha} &= \partial_i\phi_{\mu\alpha} + \partial_i\phi_{\mu\nu}\left(\Gamma^\nu\theta\right)_\alpha + \partial_i x^\nu\left(\Gamma_{\mu\nu}\theta\right)_\alpha + \tfrac{1}{6}\left(\Gamma_{\mu\nu}\theta\right)_\alpha\bar\theta\Gamma^\nu\partial_i\theta + \tfrac{1}{6}\left(\Gamma^\nu\theta\right)_\alpha\bar\theta\Gamma_{\mu\nu}\partial_i\theta \ , \\
L_{i\alpha\beta} &= \partial_i\phi_{\alpha\beta} - \tfrac{1}{2}x^\mu\partial_i\phi_{\mu\nu}(\Gamma^\nu)_{\alpha\beta} + \partial_i\phi_{\mu\nu}\left(\Gamma^\mu\theta\right)_{(\alpha}\left(\Gamma^\nu\theta\right)_{\beta)} + \tfrac{1}{4}\left(\bar\theta\partial_i\phi_\mu\right)\left(\Gamma^\mu\right)_{\alpha\beta} \\
&\quad +2\left(\Gamma^\mu\theta\right)_{(\alpha}\partial_i\phi_{\mu\beta)} - \tfrac{1}{2}x^\mu\partial_i x^\nu(\Gamma_{\mu\nu})_{\alpha\beta} - \left(\Gamma^\nu\theta\right)_{(\alpha}\left(\Gamma_{\mu\nu}\theta\right)_{\beta)}\partial_i x^\mu \\
&\quad -\tfrac{1}{12}\left(\Gamma_\nu\theta\right)_{(\alpha}\left(\Gamma^{\mu\nu}\theta\right)_{\beta)}\left(\bar\theta\Gamma_\mu\partial_i\theta\right) - \tfrac{1}{12}\left(\Gamma_\nu\theta\right)_{(\alpha}\left(\Gamma_\mu\theta\right)_{\beta)}\left(\bar\theta\Gamma^{\mu\nu}\partial_i\theta\right) \ . \qquad (2.10)
\end{aligned}
$$

Just as in the string case [4], since the Σ generators (anti) commute with each other, one can consistently impose the physical state condition $\Sigma\ \Phi(Z) = 0$, leading to superfields $\Phi(x,\theta)$ that depend only on the coordinates of the ordinary superspace (x^μ, θ^α). Of course, there may be subtleties in imposing these conditions which may arise from global considerations which may lead to drastically different quantization schemes. We leave these questions for a future investigation.

3 Reformulation of the $D = 11$ Supermembrane Action

The usual formulation of the $D = 11$ supermembrane action, as well as its reformulation (in flat as well as curved superspace) takes the following universal form [2]

$$
\begin{aligned}
I = \int d^3\sigma \quad & \Big[-\tfrac{1}{2}\sqrt{-\gamma}\gamma^{ij}\left(\partial_i Z^M L_M{}^a\right)\left(\partial_i Z^M L_{Ma}\right) + \tfrac{1}{2}\sqrt{-\gamma} \\
& -\epsilon^{ijk}\partial_i Z^M\partial_j Z^N\partial_k Z^P B_{PNM}\Big] \ , \qquad (3.1)
\end{aligned}
$$

where $a = 0, 1, ..., 10$ is the tangent space Lorentz vector index, γ_{ij} is the worldvolume metric and $\gamma = \det\gamma_{ij}$. The superspace coordinates Z^M and the super three-form B_{MNP} have to be defined in each formulation. In the usual formulation, the superspace coordinates are $Z^M = (x^\mu, \theta^\alpha)$, corresponding to the ordinary $D = 11$ super-Poincaré algebra, and B is defined in such a way that its field strength $H = dB$ satisfies certain superspace constraints which can be found in [2]. These constraints, along with super torsion constraints ensure the κ–symmetry of the action. In flat superspace, H takes the form [2]

$$
H = L^\mu \wedge L^\nu \wedge L^\alpha \wedge L^\beta\, \Gamma_{\mu\nu\alpha\beta} \ . \qquad (3.2)
$$

One can show that $dH = 0$, by using the first two equations in (2.8 Extension of the $D = 11$ Super Poincaré Algebraequation.2.8) and the Γ–matrix identity (2.2 Extension of the $D = 11$ Super Poincaré Algebraequation.2.2). Furthermore, one can solve for B as follows [21]:

$$
\begin{aligned}
B_{\rm old} = \quad & L^\mu \wedge L^\nu \wedge L^\alpha(\Gamma_{\mu\nu}\theta)_\alpha - \tfrac{1}{2}L^\mu \wedge L^\alpha \wedge L^\beta(\Gamma_{\mu\nu}\theta)_\alpha(\Gamma^\nu\theta)_\beta \\
& -\tfrac{1}{12}L^\alpha \wedge L^\beta \wedge L^\gamma(\Gamma_{\mu\nu}\theta)_\alpha(\Gamma^\mu\theta)_\beta(\Gamma^\nu)\theta)_\gamma \ . \qquad (3.3)
\end{aligned}
$$

In the new formulation of the supermembrane [3], the coordinates Z^M now refer to those defined in (2.6 Extension of the $D=11$ Super Poincaré Algebraequation.2.6). In order to maintain the κ symmetry, the field strength $H = dB$ should still take the form (3.2 Reformulation of the $D = 11$ Supermembrane Actionequation.3.2). However, the Bianchi identity $dH = 0$ must now be satisfied in the full supergroup whose algebra is given in (2.1 Extension of the $D = 11$ Super Poincaré Algebraequation.2.1). Indeed, using the structure equations of the full group as given in (2.8 Extension of the $D = 11$ Super Poincaré Algebraequation.2.8), we have shown that $dH = 0$, and that the corresponding super three-form takes the following G_L invariant form [3]:

$$B_{\rm new} = \tfrac{2}{3} L^\mu \wedge L^\nu \wedge L_{\mu\nu} + \tfrac{3}{5} L^\mu \wedge L^\alpha \wedge L_{\mu\alpha} - \tfrac{2}{15} L^\alpha \wedge L^\beta \wedge L_{\alpha\beta}\ . \tag{3.4}$$

The fact that H takes the same form in both formulation means that the dependence on all of the ϕ coordinates associated with the Σ charges is contained in total derivative terms. An interesting fact is that, the central generators $\Sigma^{\alpha\beta}$ are essential for this phenomenon to happen. Indeed, one can show that the last term in (3.4 Reformulation of the $D = 11$ Supermembrane Actionequation.3.4) is necessary for $H = dB$ to take the required form (3.2 Reformulation of the $D = 11$ Supermembrane Actionequation.3.2). All these results have stringy analogs, as discovered by Siegel [4].

The action (3.1 Reformulation of the $D = 11$ Supermembrane Actionequation.3.1) is manifestly invariant under the global G_L transformations, which include the supersymmetry transformations [3]. There is a Noether current associated with this global symmetry. The algebra of the corresponding charge densities contains field dependent extensions, as expected. The action is also invariant under the local κ–symmetry transformations

$$\delta Z^M = \kappa^\alpha (1+\Gamma)_\alpha{}^\beta L_\beta{}^M\ , \tag{3.5}$$

where Z^M are the full superspace coordinates (see (2.6 Extension of the $D = 11$ Super Poincaré Algebraequation.2.6)), and Γ is defined by

$$\Gamma = \frac{1}{3!\sqrt{-\gamma}} \epsilon^{ijk} L_i^\mu L_j^\nu L_k^\rho \Gamma_{\mu\nu\rho}\ . \tag{3.6}$$

One might consider the possibility of using a closed four-form that would differ from (3.2 Reformulation of the $D = 11$ Supermembrane Actionequation.3.2) by containing the left-invariant one-forms other than L^μ and L^α. Indeed, we have found two simple such forms, which we denote by H' and H''. They are given by

$$\begin{aligned} H' &= L^\alpha \wedge L^\beta \wedge L_{\mu\nu} \wedge L^\nu\, \Gamma^\mu_{\alpha\beta}\ , && (3.7)\\ H'' &= L^\alpha \wedge L^\beta \wedge L_{\mu\gamma} \wedge L^\gamma\, \Gamma^\mu_{\alpha\beta}\ . && (3.8) \end{aligned}$$

Writing $H' = dB'$ and $H'' = dB''$, one finds that

$$
\begin{aligned}
B' &= -\tfrac{2}{3} L^\mu \wedge L^\nu \wedge L_{\mu\nu} + \tfrac{3}{10} L^\mu \wedge L^\alpha \wedge L_{\mu\alpha} - \tfrac{1}{15} L^\alpha \wedge L^\beta \wedge L_{\alpha\beta} \ , \qquad (3.9) \\
B'' &= \tfrac{1}{5} L^\mu \wedge L^\alpha \wedge L_{\mu\alpha} + \tfrac{2}{5} L^\alpha \wedge L^\beta \wedge L_{\alpha\beta} \ . \qquad (3.10)
\end{aligned}
$$

We can use a combination of B_{new}, B' and B'' to construct a new supermembrane action. If the combination used contains $L_{\mu\nu}$ and/or $L_{\mu\alpha}$, then we need to introduce kinetic terms for the new coordinates $\phi_{\mu\nu}$ and/or $\phi_{\mu\alpha}$. It is a nontrivial matter to achieve κ symmetry, or its analogs, in such actions. Ideally, one would like to find such symmetries to gauge away the unwanted degrees of freedom, and to arrive at an anomaly-free consistent theory without ghosts and tachyons.

Interestingly enough, Curtright [7] did consider a superstring theory in eleven dimensions which contained the extra coordinates $\phi_{\mu\nu}$. Although, it is not clear how to achieve in this model the properties mentioned above, it is nonetheless interesting to see how it can be reformulated in our geometrical framework, based on the extended super-Poincaré algebra that contains both, the extra fermionic charge and the super two-form charges. We now turn to a description of this model.

4 Superstring in Eleven Dimensions?

Let us assume that a Green-Shwarz type action for superstring in eleven dimensions consist of a kinetic term and a Wess-Zumino term. The latter would require the existence of a closed super three-form in target superspace. Given the ingredients of the geometrical framework described in the previuos sections, we see that indeed such a form exists:

$$
H_3 = L^\alpha \wedge L^\beta \wedge \Big(L^\mu (\Gamma_\mu)_{\alpha\beta} - \tfrac{1}{10} L_{\mu\nu} \Gamma^{\mu\nu}_{\alpha\beta} \Big) \ . \qquad (4.1)
$$

This form is closed, due to the Γ–matrix identities (2.2 Extension of the $D = 11$ Super Poincaré Algebraequation.2.2). In fact, it is easy to see that we can write $H_3 = dB_2$, with

$$
B_2 = L_\alpha \wedge L^\alpha \ . \qquad (4.2)
$$

Since H_3 depends on $L_{\mu\nu}$, we need to introduce a kinetic term for the coordinates $\phi_{\mu\nu}$. The simplest choice for a manifestly supersymmetric action is then

$$
I = \int d^2\sigma \Big[-\tfrac{1}{2} \sqrt{-\gamma} \gamma^{ij} \left(L_i^\mu L_{j\mu} + L_i^{\mu\nu} L_{j\mu\nu} \right) - \epsilon^{ij} \partial_i Z^M \partial_j Z^N \, B_{NM} \Big] \ , \qquad (4.3)
$$

where $Z^M = (x^\mu, \theta^\alpha, \phi_\alpha, \phi_{\mu\nu})$, L_i^μ and $L_i^{\mu\nu}$ are defined in (2.10 Extension of the $D = 11$ Super Poincaré Algebraequation.2.10) and B_{NM} are the components of the super two-form B_2 defined in (4.2Superstring in Eleven Dimensions?equation.4.2). Dropping the total derivative term that

contains the coordinate ϕ_α, the action (4.3Superstring in Eleven Dimensions?equation.4.3) reduces to

$$I = \int d^2\sigma \Big[-\tfrac{1}{2}\sqrt{-\gamma}\gamma^{ij}\,(L_i^\mu L_{j\mu} + L_i^{\mu\nu} L_{j\mu\nu}) + \epsilon^{ij}\bar\theta(L_i^\mu \Gamma_\mu - \tfrac{1}{10}L_i^{\mu\nu}\Gamma_{\mu\nu})\partial_j\theta \Big]\ . \tag{4.4}$$

The Nambu-Goto version of this action where the kinetic terms are replaced by $\sqrt{-\gamma}$ with $\gamma_{ij} = L_i^\mu L_{j\mu} - \frac{1}{10}L_i^{\mu\nu}L_{j\mu\nu}$ was proposed long ago by Curtright [7]. Considering local fermionic transformations of the form $\delta Z^M = \kappa^\alpha L_\alpha{}^M$, one finds that invariance of the action under these transformations imposes the condition, $P_-^{ij}(L_j^\mu\Gamma_\mu - \frac{1}{10}L_j^{\mu\nu}\Gamma_{\mu\nu})\kappa = 0$, where $P_-^{ij} = (\gamma^{ij} - \epsilon^{ij}/\sqrt{-\gamma})$. This is a very stringent condition on the parameter κ, and we can find no solution in eleven dimensions. Furthemore, as discussed in [7], the physical significance of this action is not clear. It is conceivable that new kinds of fermionic and bosonic local symmetries that generalize the κ–symmetry exist in the enlarged superspace and that they are crucial in determining the true degrees of freedom and in finding a physically viable model. To find such symmetries, a better geometrical understanding of the κ–symmetry is needed.

Acknowledgements

I thank E. Bergshoeff, T. Curtright, M.J. Duff, M. Günaydın and W. Siegel for discussions.

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